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“Precise and fast computation of generalized Fermi-Dirac integral by  
parameter polynomial approximation”

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The generalized Fermi-Dirac integral,  $F_k(\eta, \beta)$ , is approximated by a group of polynomials of  $\beta$  as  $F_k(\eta, \beta) \approx \sum_{j=0}^J g_j \beta^j F_{k+j}(\eta)$  where  $J = 1(1)10$ . Here  $F_k(\eta)$  is the Fermi-Dirac integral of order  $k$  while  $g_j$  are the numerical coefficients of the single and double precision minimax polynomial approximations of the generalization factor as  $\sqrt{1+x/2} \approx \sum_{j=0}^J g_j x^j$ . If  $\beta$  is not so large, an appropriate combination of these approximations computes  $F_k(\eta, \beta)$  precisely when  $\eta$  is too small to apply the optimally truncated Sommerfeld expansion (Fukushima, 2014, Appl. Math. Comp., 234, 417). For example, a degree 8 single precision polynomial approximation guarantees the 24 bit accuracy of  $F_k(\eta, \beta)$  of the orders,  $k = -1/2(1)5/2$ , when  $-\infty < \eta \leq 8.92$  and  $\beta \leq 0.2113$ . Also, a degree 7 double precision polynomial approximation assures the 15 digit accuracy of  $F_k(\eta, \beta)$  of the same orders when  $-\infty < \eta \leq 29.33$  and  $0 \leq \beta \leq 3.999 \times 10^{-3}$ . Thanks to the piecewise minimax rational approximations of  $F_k(\eta)$  (Fukushima, 2015, Appl. Math. Comp., 259, 708), the averaged CPU time of the new method is only 0.9–1.4 times that of the evaluation of the integrand of  $F_k(\eta, \beta)$ . Since most of  $F_k(\eta)$  are commonly used in the approximation of  $F_k(\eta, \beta)$  of multiple contiguous orders, the simultaneous computation of  $F_k(\eta, \beta)$  of these orders is further accelerated by the factor 2–4. As a result, the new method runs 70–450 times faster than the direct numerical integration in practical applications requiring  $F_k(\eta, \beta)$ .